

# Abstract Algebra summary

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## 1 Recap on sets, functions and relations

1.1 Standard notation

1.2 Functions; restriction, associativity of composition

1.3 Images and pre-images; properties

1.4 Checking a function is well-defined

1.5 Equivalence relations and classes; partitioning

1.6 Introduction to groups

## 2 Permutation groups

2.1 Permutations; grouphood of  $\text{Sym}(S)$

2.2 Cycle decomposition (existence and uniqueness); orbits

2.3 Examples. Isomorphism of  $D_3$  to  $S_3$

2.4 Order of a permutation; order as LCM of cycle lengths

2.5 Parity, with well-definedness; elements of  $A_7$

2.6 Conjugates of permutations. Conjugacy  $\Leftrightarrow$  same type.

## 3 Groups

3.1 Definitions and notation

3.2 Basic consequences of axioms:

- Uniqueness of identity and inverse
- Generators, cyclic groups, order of elements
- $g^r = e \Rightarrow o(g) | r$
- Two facts about finite cyclic groups
- Conjugate elements and conjugacy classes. Example:  $D_6$

- 3.3 Isomorphisms
  - Preservation of identity and inverse
  - Isomorphism of cyclic groups to  $\mathbf{Z}$
- 3.4 Creating new groups. Intersections
- 3.5 Subgroup lattices and partial orderings
- 4 Cyclic groups
  - 4.1 Basic theory. Isomorphism to  $\mathbf{Z}$ , cyclicity of subgroups
  - 4.2 Infinite cyclic groups; adding same. Prime multiplication groups
  - 4.3 Finite cyclic groups. Properties.  $\mathbf{Z}_{pq} \cong \mathbf{Z}_p \times \mathbf{Z}_q$
- 5 Lagrange's theorem
  - 5.1 Statement. Consequence (prime order groups are cyclic)
  - 5.2 Cosets
  - 5.3 Properties of cosets: other defn., partitioning, bijection
  - 5.4 groups of order  $\leq 7$ . 4 and 6 in gory detail.
- 6 Normal subgroups and quotient groups
  - 6.1 Normal subgroups. Equivalent definitions.  $H \triangleleft G$  if  $G$  contains 2 cosets of  $H$ .
  - 6.2 Quotient groups
  - 6.3 Examples of same
- 7 Homomorphisms
  - 7.1 Basic facts. Kernel (is a normal sg, and converse) and image (sg)
  - 7.2 Examples
  - 7.3 Isomorphism theorem
  - 7.4 Examples:  $\text{Aut}(S_3) \cong S_3$ , homs from  $S_4$  to  $A_4$ ,  $S_3$
  - 7.5 Quotient groups via congruences
  - 7.6 Epilogue